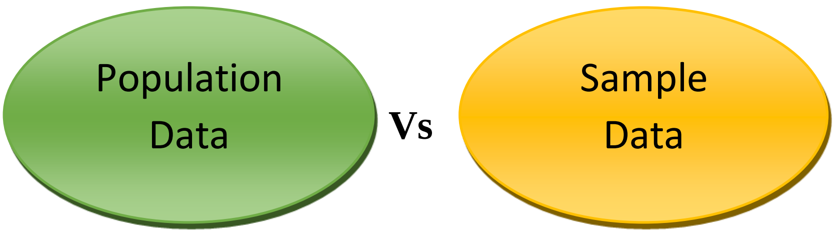
PROBABILITY DISTRIBUTION

A distribution represent the possible values a random variable can take and how often they occur.

Mean – it represent the average value which is denoted by μ (Meu) and measured in seconds

Variance – it represent how spread out the data is, denoted by σ^2 (Sigma Square). It is pertinent to note that it cannot be measured in seconds square which make no sense,therefore, variance is measured by ***Standard Deviation*** which is the square root of variance $\sqrt{\sigma^2}$andhas the same unit as means.

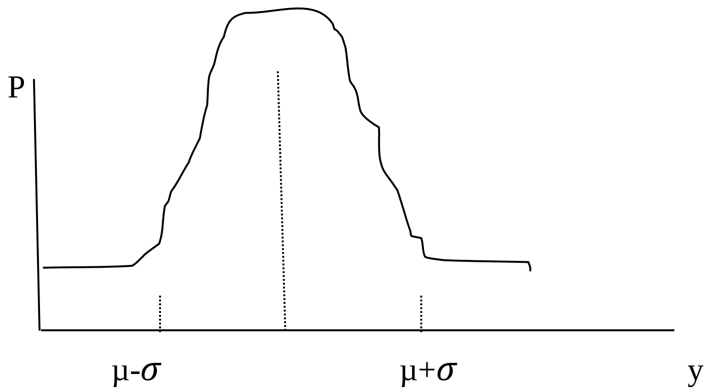
There are two kinds of data i.e. population data and sample data.



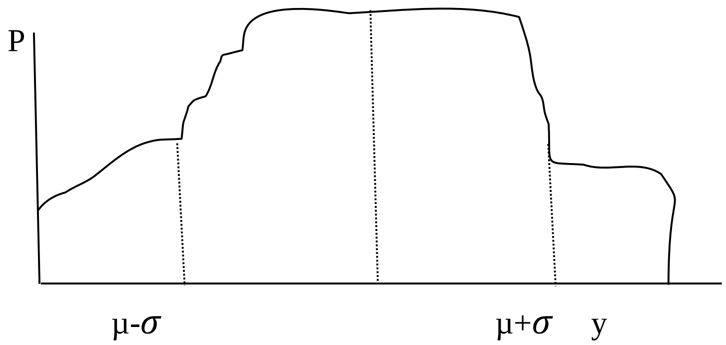
“all the data” **“just a part of it”**

	Population Data	Sample Data
Mean	μ	$\bar{x}$
Variance	σ^2	S^2
Standard Deviation	σ	S

The more overfilled the mid of the distribution, the more data falls within that interval as show in figure



The fewer data falls within the interval, the more spread the data is, as shown in figure



Notation of Distributions:

Y – Actual outcome

y – one of the possible outcomes

P(Y=y) – Probability distribution which is equal to p(y)

TYPES OF DISTRIBUTIONS:

Two major kind of distributions based on the type of likely values for the variables are,

A. Discrete Distributions

B. Continuous Distributions

COMPARISON BETWEEN DISCRETE AND CONTINUOUS DISTRIBUTIONS:

Discrete Distributions	Continuous Distribution
Discrete distributions have finite number of different possible outcomes	Continuous distributions have infinite many consecutive possible values
We can add up individual values to find out the probability of an interval	We cannot add up individual values to find out the probability of an interval because there are many of them
Discrete distributions can be expressed with a graph, piece-wise function or table	Continuous distributions can be expressed with a continuous function or graph
In discrete distributions, graph consists of bars lined up one after the other	In continuous distributions, graph consists of a smooth curve
Expected values might be unachievable	To calculate the chance of an interval, we required integrals

Notation Explanation:

$X \sim N(\mu, \sigma^2)$

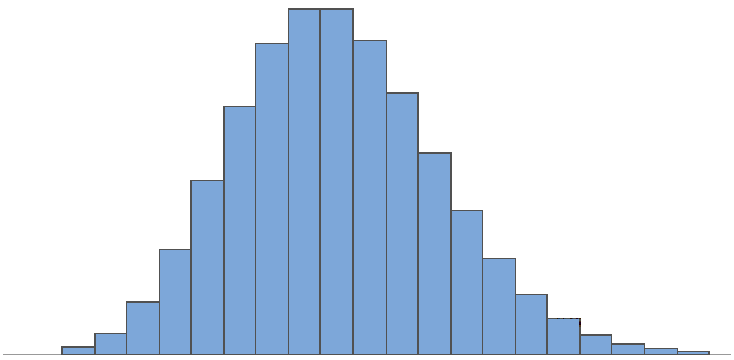
Here, X is variable, ~ tilde, N is types of distribution and (μ, σ^2) are its characteristics

A. DISCRETE DISTRIBUTIONS:

Discrete distributions have finite number of different possible outcomes. Its main characteristics are given below:-

- We can add up individual values to find out the probability of an interval
- Discrete distributions can be expressed with a graph, piece-wise function or table
- In discrete distributions, graph consists of bars lined up one after the other
- Expected values might be unachievable
- $P(Y \leq y) = P(Y < y + 1)$

In graph, the discrete distributions are looks like as,



Examples of Discrete Distributions:

i. Bernoulli Distribution
ii. Binomial Distribution
iii. Uniform Distribution
iv. Poisson Distribution

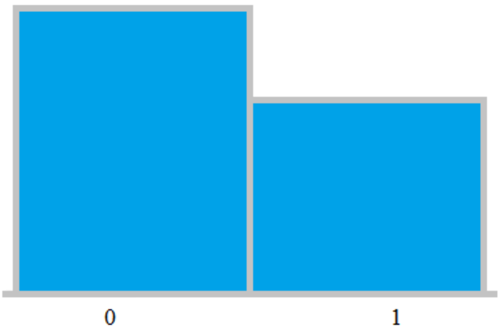
i. Bernoulli Distribution:

In Bernoulli distribution there is only one trial and only two possible outcomes i.e. success or failure. It is denoted by y ~Bern(p). The main characteristics of Bernoulli distributions are:

- It consists of a single trial
- Two possible outcomes
- $E(Y) = p$
- $Var(Y) = p \times (1 - p)$

Examples and Uses:

- Guessing a single True/False question
- It is mostly used when trying to find out what we expect to obtain a single trial of an experiment.



ii. Binomial Distribution:

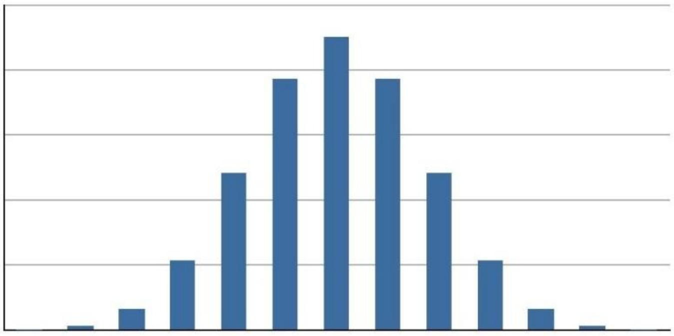
A sequence of identical Bernoulli events is called Binomial and follows a Binomial distribution. It is denoted by Y ~ B(n, p). The main characteristics of Binomial distribution are:

- Over the n trials, it measures the frequency of occurrence of one of the possible result.
- $E(Y) = n \times p$
- $P(Y = y) = C(y, n) \times p^{y} \times (1 - p)^{n-y}$
- $Var(Y) = n \times p \times (1 - p)$

Examples and Uses:

- Simply determine, how many times we obtain a head if we flip a coin 10 times.

- It is mostly used when we try to predict how likelihood an event occur over a series of trials.



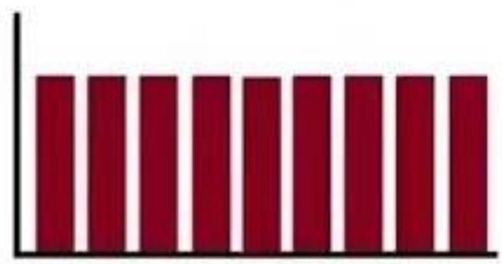
iii. Uniform Distribution:

In uniform distribution all the outcomes are equally likely. It is denoted by Y ~ U(a, b). If the values are categorical, we simply indicate the number of categories, like Y ~ U(a). The main characteristics of Uniform Distribution are:

- In uniform distribution all the outcomes are equally likely.
- In graph, all the bars are equally tall
- The expected value and variance have no predictive power

Examples and Uses:

- Result obtained after rolling a die
- Due to its equality, it is mostly used in shuffling algorithms



iv. Poisson Distribution:

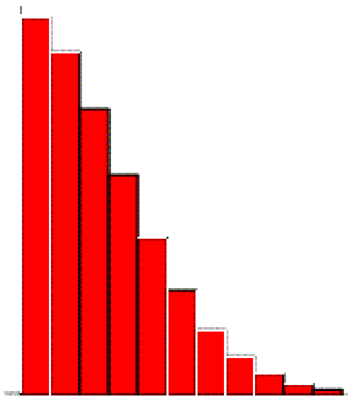
Poisson distribution is used to determine how likelihood a certain event occur over a given interval of time or distance. It is denoted by Y ~ Po(λ). The main characteristics of poisson distribution are:

- It measures the frequency over an interval of time or distance.
- $E(Y) = \lambda$
- $P(Y = y) = \frac{\lambda^y}{\lambda! e^{-\lambda}}$

- Var(Y) = λ

Examples and Uses:

- It is used to determine how likelihood a certain event occur over a given interval of time or distance.
- Mostly used in marketing analysis to find out whether more than average visits are out of the ordinary or otherwise.

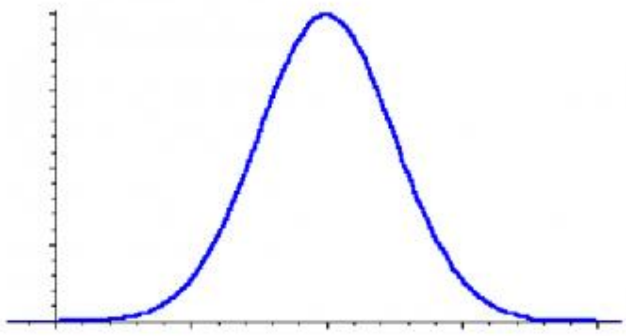


B. CONTINUOUS DISTRIBUTIONS:

Continuous distributions have infinite many consecutive possible values. Its main characteristics are given below:-

- We cannot add up individual values to find out the probability of an interval because there are many of them
- Continuous distributions can be expressed with a continuous function or graph
- In continuous distributions, graph consists of a smooth curve
- To calculate the chance of an interval, we required integrals
- P(Y = y) = 0 for any distinct value y.
- P(Y<y) = P(Y $\leq$ y)

In graph, the continuous distributions are looks like as,



Examples of Continuous Distributions:

i. Normal Distribution
ii. Chi-Squared Distribution
iii. Exponential Distribution
iv. Logistic Distribution
v. Students' T Distribution

i. Normal Distribution:

It shows a distribution that most natural events follow. It is denoted by Y ~ (μ,σ^2). The main characteristics of normal distribution are:

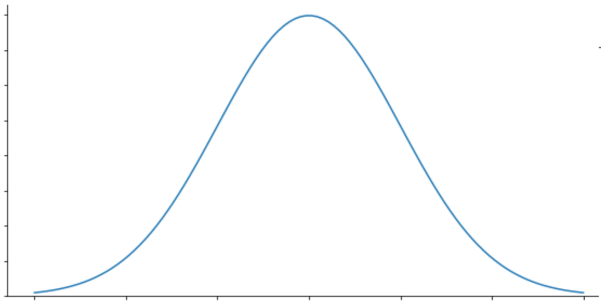
- Graph obtained from normal distribution is bell-shaped curve, symmetric and has shrill tails.
- 68% of all its all values should fall in the interval, i.e. ($\mu - \sigma$, $\mu + \sigma$)
- $E(Y) = \mu$
- $Var(Y) = \sigma^2$

Examples and Uses:

- Normal distributions are mostly observed in the size of animals in the desert.
- We can convert any normal distribution into a standard normal distribution. Normal distribution could be standardized to use the Z-table

  i.e. $z = \frac{y - \mu}{\sigma}$

  Where, σ ensures standard deviation is 1and μ ensures mean is 0.



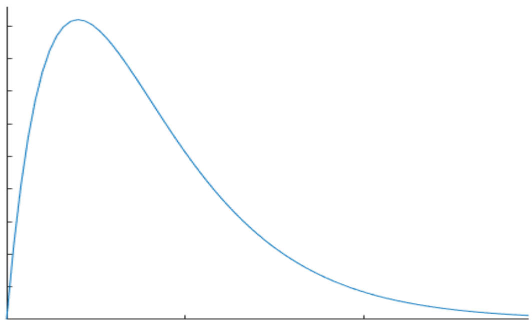
ii. Chi-Squared Distribution:

Chi-Squared distribution is frequently being used. It is mostly used to test wow of fit. It is denoted by Y ~ X^2 (k). The main characteristics of Chi-Squared distribution are:

- The graph obtained from Chi-Squared distribution is asymmetric and skewed to the right.
- It is square of the t-distribution.
- E(Y) = k
- Var(Y) = 2k

Examples and Uses:

- It is mostly used to test wow of fit.
- It comprises a table of known values for its CDF called the x^2 – table.



iii. Exponential Distribution:

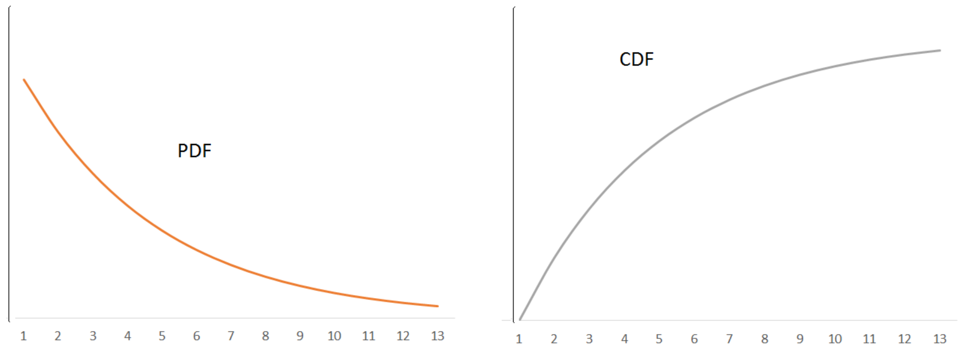
It is usually observed in events which considerably change early on. It is denoted by Y ~ Exp(λ). The main characteristics of exponential distribution are:

- Probability and Cumulative Distributed Functions (PDF & CDF) plateau after a certain point.

- We do not have a table to known the values like the Normal or Chi-Squared Distributions, therefore, we mostly used natural logarithm to change the values of exponential distributions.
- E(Y) = $\frac{1}{\lambda}$
- Var(Y) = = $\frac{1}{\lambda^2}$

Examples and Uses:

- It is mostly used with dynamically changing variables, such as online websites traffic



iv. Logistic Distribution:

It is used to observe how continuous variable inputs can affect the probability of a binary result. It is denoted by Y ~ Logistic(μ, s). The main characteristics of logistic distribution are:

- The Cumulative Distributed Function picks up when we reach values near the mean.
- The lesser the scale parameter, the faster it reaches values close to 1.
- E(Y) = μ
- Var(Y) = $\frac{s^2 \times \pi^2}{3}$

Examples and Uses:

- It is mostly used in sports to predict how a player's or team's feat can conclude the result of the match.

v. Students' T Distribution

Students' T Distribution or simply called T Distribution is used to estimate population limitation when the sample size is small and population variance is not known. It is denoted by Y~ t(k). The main characteristics of Students' T Distribution are:

- A small sample size estimation of a normal distribution
- Its graph is symmetric and bell-shaped curve, however, it has large tails.
- If $k > 1$ then $E(Y) = \mu$ and $Var(Y) = s^2 \times \frac{k}{k-2}$

Examples and Uses:

- It is used in examination of a small sample data which normally follows a normal distribution.